Lecture 7

Bipolar Junction Transistors (BJTs)

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Overview

• Reading
  – S&G: Chapter 2.5, 3.2, 6.1~3

• Supplemental Reading
  – Sedra&Smith: Chapter 4.1~3

• Background
  – We will begin looking at transistors with the bipolar junction transistor. We will spend some time understanding how they work based on what we know about pn junctions. One way to look at a BJT transistor is two back-to-back diodes, has very different characteristics.
Bipolar Junction Transistor

- NPN BJT shown
- 3 terminals: emitter, base, and collector
- 2 junctions: emitter-base junction and collector-base junction
- Depending on the biasing across each of the junctions, different modes of operation are obtained – cutoff, active, and saturation

<table>
<thead>
<tr>
<th>MODE</th>
<th>EBJ</th>
<th>CBJ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cutoff</td>
<td>Reverse</td>
<td>Reverse</td>
</tr>
<tr>
<td>Active</td>
<td>Forward</td>
<td>Reverse</td>
</tr>
<tr>
<td>Saturation</td>
<td>Forward</td>
<td>Forward</td>
</tr>
</tbody>
</table>
BJT in Active Mode

- Two external voltage sources set the bias conditions for active mode
  - Note EBJ is forward biased and CBJ is reverse biased
- Operation
  - Forward bias of EBJ injects electrons from emitter into base (small number of holes injected from base into emitter)
  - Most electrons shoot through the base into the collector across the reverse bias junction (think about band diagram)
  - Some electrons recombine with majority carrier in p-type base
Minority Carrier Concentration Profiles

- Current dominated by electrons from emitter to base (by design) b/c of the forward bias and minority carrier concentration gradient (diffusion) through the base
  - some recombination causes bowing of electron concentration
  - base is designed to be fairly short (minimize recombination)
  - emitter is heavily (sometimes degenerately) doped and base is lightly doped
- Drift currents are usually small and neglected
Diffusion Current Through the Base

- Diffusion of electrons through the base is set by concentration profile at the EBJ
  \[ n_p(0) = n_{p0} e^{v_{BE}/V_T} \]
- Diffusion current of electrons through the base is (assuming an ideal straight line case):
  \[ I_n = A_E q D_n \frac{dn_p(x)}{dx} = A_E q D_n \left( -\frac{n_p(0)}{W} \right) \]
- Due to recombination in the base, the current at the EBJ and current at the CBJ are not equal and differ by a base current
Collector Current

- Electrons that diffuse across the base to the CBJ junction are swept across the CBJ depletion to the collector b/c of the higher potential applied to the collector.

\[ i_C = I_s e^{v_{BE}/V_T} \]

where the saturation current is

\[ I_S = qA_E D_n n_{p0}/W \]

and we can rewrite the saturation current as:

\[ I_S = \frac{qA_E D_n n_i^2}{N_A W} \]

- Note that \( i_C \) is independent of \( v_{CB} \) (potential bias across CBJ) ideally
- Saturation current is
  - inversely proportional to \( W \) and directly proportional to \( A_E \)
    - Want short base and large emitter area for high currents
  - dependent on temperature given \( n_i^2 \)
Base Current

• Base current $i_B$ composed of two components:
  – holes injected from the base region into the emitter region
    $$i_{B1} = \frac{qA_{E}D_{p}n_{i}^{2}}{N_D L_P} e^{v_{BE}/V_T}$$
  – holes supplied due to recombination in the base with diffusing electrons and depends on minority carrier lifetime $\tau_b$ in the base
    $$i_{B2} = \frac{Q_n}{\tau_b}$$

And the Q in the base is

$$Q_n = \frac{qA_{E}Wn_{i}^{2}}{N_A} e^{v_{BE}/V_T}$$

So, current is

$$i_{B2} = \frac{qA_{E}Wn_{i}^{2}}{N_A \tau_b} e^{v_{BE}/V_T}$$

• Total base current is

$$i_B = \left( \frac{qA_{E}D_{p}n_{i}^{2}}{N_D L_P} + \frac{qA_{E}Wn_{i}^{2}}{N_A \tau_b} \right) e^{v_{BE}/V_T}$$
Beta

• Can relate $i_B$ and $i_C$ by the following equation

\[ i_B = \frac{i_C}{\beta} = \frac{I_S}{\beta} e^{v_{BE}/V_T} \]

and $\beta$ is

\[ \beta = \frac{1}{\frac{D_p}{D_n} \frac{N_A}{N_D} \frac{W}{L_p} + \frac{1}{2} \frac{W^2}{D_n \tau_b}} \]

– Beta is constant for a particular transistor
– On the order of 100-200 in modern devices (but can be higher)
– Called the common-emitter current gain

• For high current gain, want small $W$, low $N_A$, high $N_D$
Emitter Current

- Emitter current is the sum of $i_C$ and $i_B$

\[ i_E = i_C + i_B \]

\[ i_E = \frac{\beta + 1}{\beta} i_C \]

\[ i_C = \alpha i_E \text{ where } \alpha = \frac{\beta}{\beta + 1} \]

$\alpha$ is called the common-base current gain
BJT Equivalent Circuits
Circuit Symbols and Conventions

- BJTs are not symmetric devices
  - doping and physical dimensions are different for emitter and collector
I-V Characteristics

\[ I_C = I_S e^{V_{BE}/V_T} \]

- Collector current vs. \( v_{CB} \) shows the BJT looks like a current source (ideally)
  - Plot only shows values where BCJ is reverse biased and so BJT in active region
- However, real BJTs have non-ideal effects
Early Effect

\[ I_C = I_S e^{V_{BE}/V_T} \left(1 + \frac{V_{CE}}{V_A}\right) \]

- **Early Effect**
  - Current in active region depends (slightly) on \( V_{CE} \)
  - \( V_A \) is a parameter for the BJT (50 to 100) and called the Early voltage
  - Due to a decrease in effective base width \( W \) as reverse bias increases
  - Account for Early effect with additional term in collector current equation
  - Nonzero slope means the output resistance is NOT infinite, but…
    - \( I_C \) is collector current at the boundary of active region

\[ r_o \approx \frac{V_A}{I_C} \]
Early Effect Cont’d

• What causes the Early Effect?
  – Increasing $V_{CB}$ causes depletion region of CBJ to grow and so the effective base width decreases (base-width modulation)
  – Shorter effective base width $\Rightarrow$ higher $dn/dx$

$$I_n = A_E q D_n \frac{dn_p(x)}{dx} = A_E q D_n \left( -\frac{n_p(0)}{W} \right)$$
BJT DC Analysis

• Use the simple constant-$V_{BE}$ model
  – Assume $V_{BE} = 0.7$-V regardless of exact current value (reasonable b/c of exponential relationship)
• Make sure the BJT current equations and region of operation match
  – So far, we only have equations for the active region
• Utilize the relationships ($\beta$ and $\alpha$) between collector, base, and emitter currents to solve for all currents
• Example:
BJT Amplifier

- To operate as an amplifier, the BJT must be biased to operate in active mode and then superimpose a small voltage signal $v_{be}$ to the base.
- Under DC conditions,

\[
\begin{align*}
I_C &= I_S e^{V_{BE}/V_T} \\
I_E &= I_C/\alpha \\
I_B &= I_C/\beta \\
V_C &= V_{CE} = V_{CC} - I_C R_C
\end{align*}
\]
• The DC condition biases the BJT to the point $Q$ on the plot.
• Adding a small voltage signal $v_{be}$ translates into a current signal that we can write as
  
  $$i_C = I_S e^{v_{BE}/V_T} = I_S e^{(V_{BE}+v_{be})/V_T}$$
  
  $$= I_S e^{V_{BE}/V_T} = I_C e^{v_{be}/V_T}$$

• If $v_{be} \ll V_T$
  
  $$i_C = I_C \left( 1 + \frac{v_{be}}{V_T} \right)$$

• The collector current has two components: $I_C$ and $i_c$ and we can rewrite the small signal current as
  
  $$i_c = \frac{I_C}{V_T} v_{be} = g_m v_{be}$$  where  $$g_m = \left. \frac{\partial i_C}{\partial v_{BE}} \right|_{i_C=I_C}$$

  – $g_m$ is the transconductance and corresponds to the slope at $Q$
  – For small enough signals, approximate exponential curve with a linear line
Small-Signal Model

- We can model the BJT as a voltage controlled current source, but we must also account for the base current that varies with $v_{be}$

$$i_B = \frac{i_C}{\beta} = \frac{I_C}{\beta} + \frac{1}{\beta V_T} v_{be}$$

$$i_B = I_B + i_b$$

$$i_b = \frac{1}{\beta V_T} v_{be}$$

$$i_b = \frac{1}{\beta g_m v_{be}}$$

- so, the small-signal resistance looking into the base is denoted by $r_\pi$ and defined as

$$r_\pi \equiv \frac{v_{be}}{i_b} = \frac{\beta}{g_m}$$

- looking into the emitter, we get an effective small-signal resistance between base and emitter, $r_e$

$$r_e \equiv \frac{v_{be}}{i_e} = \frac{V_T}{I_E} = \frac{\alpha}{g_m} \approx \frac{1}{g_m}$$

$$r_\pi = (\beta + 1)r_e$$
To convert the voltage-controlled current source into a circuit that provides voltage gain, we connect a resistor to the collector and measure the voltage drop across it:

\[ v_C = V_{CC} - i_C R_C \]
\[ = V_{CC} - (I_C + i_c)R_C \]
\[ = (V_{CC} - I_C R_C) - i_c R_C \]
\[ v_c = V_C - i_c R_C \]
\[ v_c = -i_c R_C = -g_m v_{be} R_C \]

So, the small-signal voltage gain is:

\[ \text{Voltage Gain} \equiv \frac{v_c}{v_{be}} = -g_m R_C \]

- Remember that \( g_m \) depends on \( I_C \)

We can create an equivalent circuit to model the transistor for small signals:
- Note that this only applies for small signals \( (v_{be} < V_T) \)
Hybrid-π Model

- We can represent the small-signal model for the transistor as a voltage-controlled current source or a current-controlled current source.
- Add a resistor ($r_o$) in parallel with the dependent current source to model the Early effect.
  - From our previous example,
  \[ \text{Voltage Gain} \equiv \frac{v_c}{v_{be}} = -g_m(R_C|r_o) \]
T Model

- Sometimes, other small signal models can be more convenient to use.

\[ g_m = \frac{I_C}{V_T} \]
\[ r_e = \frac{V_T}{I_E} = \frac{\alpha}{g_m} \]
Using Small-Signal Models

• Here are the steps for using small-signal models
  1. Determine the DC operating point of the BJT
     • in particular, the collector current
  2. Calculate small-signal model parameters: $g_m$, $r_\pi$, $r_e$
  3. Eliminate DC sources
     – replace voltage sources with shorts and current sources with open circuits
  4. Replace BJT with equivalent small-signal models
     – Choose most convenient one depending on surrounding circuitry
  5. Analyze
Graphical Analysis

- Can be useful to understand the operation of BJT circuits
- First, establish DC conditions by finding $I_B$ (or $V_{BE}$)
- Second, figure out the DC operating point for $I_C$
• Apply a small signal input voltage and see $i_b$
• See how $i_b$ translates into $V_{CE}$
• Can get a feel for whether the BJT will stay in active region of operation
  – What happens if $R_C$ is larger or smaller?
Single-Stage Amplifier Configurations

• There are three basic configurations for BJT amplifiers:
  – Common-Emitter
  – Common-Base
  – Common-Collector

• We will just look at one now and then come back to the others after we’ve learned about MOSFETs
Common-Emitter Amplifier

- First, assume $R_e = 0$ (this is not $r_e$, but an explicit external resistor)
- The BJT is biased with a current source (with high output impedance) and a capacitor connects the emitter to ground
  - Cap provides an AC short at the emitter for small time-varying signals but is an open circuit for DC signals
- Can redraw the circuit with an equivalent circuit that replaces the BJT with its hybrid-$\pi$ model

\[
\frac{v_\pi}{v_s} = \frac{r_\pi}{R_s + r_\pi} \quad v_o = -g_m v_\pi \left( R_C || r_o \right) \quad \frac{v_o}{v_\pi} = -g_m \left( R_C || r_o \right) \]

\[
R_i = r_\pi \quad R_o = R_C || r_o
\]